The Project for Human Resource Development Scholarship (JDS)

Basic Mathematics Aptitude Test 2023

Solution

解答に至るための途中式は黄色のハイライト箇所です。
Part3-2 の問題は途中式が 2 パターンあります。

(Please show all your work here and write your answers in the designated space)

[PART I] Calculate the followings:

1. $3-(3-3\times(2+(5-7)))$

$3-(3-3\times(2+(5-7)))=3-(3-3\times 0)=3-3=0$

Answer : 0

2. $\frac{3}{5}\div\left(\frac{2}{5}-0.25\right)+\left(1.5\div\frac{3}{4}\right)\div\left(\frac{3}{10}-\frac{4}{5}\right)$

$\frac{3}{5}\div\left(\frac{2}{5}-0.25\right)+\left(1.5\div\frac{3}{4}\right)\div\left(\frac{3}{10}-\frac{4}{5}\right)=\frac{3}{5}\div\left(\frac{8}{20}-\frac{5}{20}\right)+\left(\frac{3}{2}\times\frac{4}{3}\right)\div\left(\frac{3}{10}-\frac{8}{10}\right)=\left(\frac{3}{5}\times\frac{20}{3}\right)+\left(\frac{4}{2}\div\left(-\frac{1}{2}\right)\right)=4-4=0$

Answer : 0

3. $\left(\sqrt{24}-\sqrt{54}\right)\times\sqrt{6}$

$\left(\sqrt{24}-\sqrt{54}\right)\times\sqrt{6}=\left(2\sqrt{6}-3\sqrt{6}\right)\times\sqrt{6}=\left(-\sqrt{6}\right)\times\sqrt{6}=-6$

Answer : -6

4. $\left(2^4\times\left(\frac{1}{4}\right)^2\right)^3\div\left(\frac{1}{4}\right)^{-2}$

$\left(2^4\times\left(\frac{1}{4}\right)^2\right)^3\div\left(\frac{1}{4}\right)^{-2}=(2^4\times 2^{-4})^3\div(2^{-2})^{-2}=(2^{4-4})^3\div 2^4=1\div 2^4=\frac{1}{16}$

Answer : $\frac{1}{16}$

(Please show all your work here and write your answers in the designated space)
[PART II] Answer the following questions:

1. Solve the following equations: $2x + 1 = 5x - 8$

$2x - 5x = -8-1 \rightarrow -3x = -9, \quad x = 3$

Answer : $x = 3$

2. Solve the following simultaneous equations for x and y.

$2x + 3y - 2 = 5$
$-x + 4y = 2$

Answer : $x = 2, \quad y = 1$

3. Find the region x satisfying the following inequality, where | | indicates the absolute value.

$$\left|\frac{3x-1}{2}\right| < 1$$

$-1 < \frac{3x-1}{2} < 1 \rightarrow -2 < 3x - 1 < 2 \rightarrow -2+1 < 3x < 2+1 \rightarrow -1 < 3x < 3$

Answer : $-\frac{1}{3} < x < 1$

4. Find the difference between the arithmetic mean and median values in the following observations x_i.

$Mean(x_i) - Median(x_i)$ where $x_i = \{14,2,7,6,3,4\}$

Arrange x_i in asceding order as $x_i = \{2,3,4,6,7,14\}$. Given six even elements, the median is the average of the two middle values of 4 and 6 and is equal to 5.
$Mean(x_i) = \frac{2+3+4+6+7+14}{6} = \frac{36}{6} = 6, \; Median(x_i) = \frac{4+6}{2} = 5 \rightarrow Mean(x_i) - Median(x_i) = 6 - 5 = 1$

Answer : 1

(Please show all your work here and write your answers in the designated space)
[PART III] Answer the following questions:

1. Solve the following equation for x.

$$\frac{4x^2}{3} - 4 = 8$$

$4x^2 = 12 \times 3 \rightarrow x^2 = (\pm 3)^2$

Answer : $x = 3$, $x = -3$ or $x = \pm 3$

2. Find the region of x satisfying the following inequality.

$$x^2 + 2x < -4x - 5$$

Solution: Both factoring and the quadratic formula can be applied to arrive at the solutions:
① $x^2 + 6x + 5 < 0 \rightarrow (x+1)(x+5) < 0 \rightarrow$ Answer: $-5 < x < -1$
② $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{(-6)^2-4\times1\times5}}{2\times1} = \frac{-6 \pm \sqrt{36-20}}{2\times1} = \frac{-6 \pm \sqrt{16}}{2\times1} = -5, -1 \rightarrow$ Answer: $-5 < x < -1$

Answer : $-5 < x < -1$

3. Solve the following equation for x.

$$2log_2(x) = log_2(6 - 5x)$$

$2log_2(x) = log_2(6-5x) \rightarrow x^2 + 5x - 6 = (x-1)(x+6) = 0$ $Since\ x > 0, x = 1.$

Answer : $x = 1$

4. Consider the following six values, [3, 2, 1, 10, 9, 5]. Suppose that the median of six values is 2^{3x}. Find the value of x.

To find the median (M) from a set of values, follow these steps:
Step 1: Arrange in ascending order: 1, 2, 3, 5, 9, 10
Step 2: Given 6 even elements, the median (M) is the average of the two middle values of 3 and 5.
Median (M) = (3 + 5) / 2 = 8 / 2 = 4
$4 = 2^2 = 2^{3x} \rightarrow 3x = 2, x = \frac{2}{3}$

Answer : $x = \frac{2}{3}$

(Please show all your work here and write your answers in the designated space)
[PART IV] Answer the following questions:

1. Determine the first-order derivative of the following. Note that e is a mathematical constant which is the base of the natural logarithm.

   $y = \frac{1}{x} + log_e x$

   $y' = -1 \times x^{-1-1} + \frac{1}{x} = -x^{-2} + \frac{1}{x} = -\frac{1}{x^2} + \frac{1}{x} = \frac{x-1}{x^2}$

   Answer : $\frac{x-1}{x^2}$

2. Find the following definite integral.

   $\int_0^3 (x + x^2)dx$

   $\int_0^3 (x + x^2)dx = \int_0^3 (x)dx + \int_0^3 (x^2)dx = \left(\frac{1}{2}x^2\right)\Big|_0^3 + \left(\frac{1}{3}x^3\right)\Big|_0^3 = \left(\frac{1}{2} \times 3^2 + \frac{1}{3} \times 3^3\right) = \frac{27}{2} = 13\frac{1}{2} = 13.5$

   Answer : $13\frac{1}{2}$ *or* 13.5

3. Let A=$\begin{bmatrix} a & -2 \\ -2 & 3 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

   Assume determinant of the matrix A is -1. Find $A^{-1}B$.

   The determinant of the matrix, denoted as $\det(A) = a \times 3 - (-2 \times -2) = -1. \rightarrow$ a=1

   $A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -2 & -1 \end{bmatrix}$ $A^{-1}B = \begin{bmatrix} -3 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ -3 & 0 \end{bmatrix}$

   Answer : $\begin{bmatrix} -4 & -1 \\ -3 & 0 \end{bmatrix}$

4. Find the value of x and y that solves the following constrained maximization problem:

   Maximize $\sqrt{2xy}$ subject to $2x + y = 10$.

   $2x + y = 10 \rightarrow y = 10 - 2x.$ $\sqrt{2xy} = \sqrt{2x(10-2x)} = \sqrt{20x - 4x^2} \rightarrow 5 - 2x = 0 \rightarrow x$ =2.5, y=5

   Answer : x =2.5 or $x = 2\frac{1}{2}$ or $x = \frac{5}{2}$ y =5

(Please show all your work here and write your answers in the designated space)
[PART V] Fill in the following blanks with correct answers.

1. Find the first derivative of the following.

$f(\theta) = \sin\left(\frac{1}{2}\theta\right)$

$f(\theta) = \sin\left(\frac{1}{2}\theta\right) \quad \rightarrow f'(\theta) = \frac{1}{2}\cos\left(\frac{1}{2}\theta\right)$

Answer : $\frac{1}{2}\cos\left(\frac{1}{2}\theta\right)$

2. Given a sequence 2^{n-1}, find the value $\sum_{n=1}^{5} 2^{n-1}$, where n is an integer.

The n[th] term of a geometric series is given as ar^{n-1}, where a ad r are first term of the geometric sequence and common ratio, respectively. Using the sum of the first *n* terms of a geometric series is given by the formula. $S_n = \frac{a(1-r^n)}{1-r}$ where r>1.

$\sum_{n=1}^{5} 2^{n-1} = \frac{1\times(1-2^5)}{1-2} = 1 + 2 + 4 + 8 + 16 = 31$

Answer : 31

3. Suppose that $\vec{a} = (3x + 6, 3)$ and $\vec{b} = (2, y)$ are vertical and that $x + y = 1$. Find x and y.

The inner product $\vec{a}\cdot\vec{b}$ must be zero, given the angle of two vectors is 90 degree (so-called orthogonal). $\vec{a}\cdot\vec{b} = 2 \times (3x + 6) + 3y = 6x + 3y + 12 = 0 \rightarrow 6x + 3y = -12$. So, we solve the simultaneous equations of $6x + 3y = -12$ and x + y=1. Thus, x=−5 y=6.

Answer : $x = -5, \quad y = 6$

4. In a firm, there are 5 computer science degree holders and 3 marketing degree holders. The chief executive officer intends to create business analysis teams by selecting two members from each group of degree holders. Find the total number of different teams that can be formed.

${}_5C_2 \times {}_3C_2 = \frac{5\times4}{2\times1} \times \frac{3\times2}{2\times1} = 10 \times 3 = 30$

Answer : 30